UN – 176

V Semester B.A./B.Sc. Examination, Nov./Dec. 2015
(Semester Scheme) (Prior to 2013-14) (OS)
MATHEMATICS – V

Time : 3 Hours

Max. Marks : 90

Instruction : *Answer **all** questions.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) In a ring $(R, +, \cdot)$, prove that $a \cdot 0 = 0 \cdot a = 0$, where 0 being the additive identity.

2) Define a commutative ring and an integral domain.

3) Define a subring of a ring and give an example.

4) Prove that the intersection of two subrings of a ring is a subring.

5) Let $(z, +, \cdot)$ be the ring of integers, define $f : z \to z$ by $f(x) = x\ \forall\ x \in z$. Show that f is homomorphism.

6) Define kernel of homomorphism.

7) If $\vec{r} = ti - t^2j + \sin t\, k$, find $\frac{d\vec{r}}{dt}$ and $\frac{d^2\vec{r}}{dt^2}$ at $t = 0$.

8) Define :
   i) Curvature
   ii) Torsion at any point for a space curve.

9) Find the unit tangent vector t for the curve $x = 3\cos t$, $y = 3\sin t$, $z = 4t$.

10) The Cartesian coordinates of a point are $(2, 2\sqrt{3}, -3)$. Find the corresponding cylindrical coordinates.

11) Find the normal vector to the cylinder $x^2 + y^2 = 16$ at $(2\sqrt{3}, 2, 0)$.

12) If $\phi(x, y, z) = x^4 + y^4 + z^4$, find $\nabla\phi$ at $(-1, 2, 3)$.

13) Find the constant 'a' so that $\vec{F} = (x + 3y)i + (y - 2z)j + (x - az)k$ is solenoidal.

P.T.O.

14) If ϕ and Ψ are two scalar point functions then prove that
$\nabla(\Psi\,\phi) = \phi\,\nabla\,\Psi + \Psi\,\nabla\,\phi$.

15) Find the divergence of $\vec{f} = 3x^2 i + 5xy^2 j + xyz^3 k$ at the point (1, 2, 3)

16) Prove that curl (grad ϕ) = 0.

17) Write Legendre's equation.

18) Show that $P_n(1) = 1$.

19) Show that $J_0^1(x) = -J_1(x)$.

20) Show that $\frac{d}{dx}[x\,J_1(x)] = xJ_0(x)$.

II. Answer **any four** of the following : **(4×5=20)**

1) Prove that every field is an integral domain. Justify your answer that converse is not true.

2) S.T. the set $S = \left\{\begin{pmatrix} a & o \\ b & c \end{pmatrix} \Big/ a, b, c \in z\right\}$ is a subring of the ring $M_2(z)$ $\forall$ 2×2 matrices over the set of integers.

3) Find all the principal ideals of the ring R = {0, 1, 2, 3, 4, 5} w.r.t. '$+_6$' and 'X_6'.

4) If $f : R \to R'$ be a homomorphism of R into R'. Then prove that f is one-one if and only if Ker f = {0}.

5) Let R be the ring of all matrices of 2×2 over z and $S = \left\{\begin{pmatrix} a & o \\ b & o \end{pmatrix} \Big/ a, b \in z\right\}$.
Show that S is a left ideal in R but not a right ideal.

6) Let I be an ideal of a ring R, then prove that :

i) R is commutative $\Rightarrow$ R/I is commutative

ii) R is a ring with unity $\Rightarrow$ R/I is a ring with unity.

III. Answer **any three** of the following : (3×5=15)

1) Derive Seret-Frenet of formulae for a space curve.

2) Find the curvature and torsion for the curve $x = u$, $y = u^2$, $z = u^3$ at $t = 1$.

3) Find the unit normal and the equation of the tangent plane to the surface $z = x^2 + y^2$, at $(1, -1, 2)$.

4) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.

5) Express the vector $\vec{f} = 3yi + x^2j - z^2k$ in cylindrical coordinates.

IV. Answer **any three** of the following : (3×5=15)

1) If $\phi(x, y, z) = x^2 y^2 z^2$ and $\vec{F} = 2xi + yj + 3zk$ find $\vec{F} \cdot \nabla\phi$ and $\vec{F} \times \nabla\phi$.

2) Show that $\vec{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational. Find a scalar field ϕ such that $f = \nabla\phi$.

3) Prove that $\text{curl}\,(\phi f) = \phi\,\text{curl} f + \text{grad}\,\phi \times f$.

4) Show that $\text{div}\,(r^n \vec{r}) = (n + 3)\, r^n$ where $r = |\vec{r}|$.

5) Prove that cylindrical coordinate system is orthogonal.

V. Answer **any two** of the following : (2×5=10)

1) Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]$ for the Legendre polynomial.

2) Show that $nP_n(x) = xP_n^1(x) - P_{n-1}^1(x)$.

3) Show that $\int_{-1}^{1} xP_n(x)\, P_{n-1}(x)\, dx = \frac{2n}{4n^2 - 1}$

OR

$J_n(x) = \frac{x}{2n}[J_{n-1}(x) + J_{n+1}(x)]$.

4) Find the solution of $xy'' + 2y' + \frac{1}{2}xy = 0$ interms of Bessel's functions.